



Examiners' Report Principal Examiner Feedback

Summer 2022

Pearson Edexcel International Advanced Level
In Further Mathematics F3 (WFM03) Paper 01

Overall, the paper seemed to be accessible for most candidates with only a few seeming to be unprepared for the examination. A few appeared to run out of time but this could have been caused by immature algebra and repeating working for some questions. Most candidates were well drilled on standard bookwork like Q1 and Q9(a).

A significant number lost marks in Q6(b) for dividing by the wrong determinant, having simplified in part (a). Question 8(c) was rarely attempted correctly and any completely correct solution was a collector's item. Question 9 had the most varied results: most got part (a) right, in part(b) the main problem was getting lost in the algebra, losing k in the x coordinate. There were few who used the sum of the roots $= -\frac{b}{a}$. Part (c) was rarely completed successfully due sometimes to errors in part (b).

Question 1

In part (a), nearly all candidates started with the LHS and substituted in the exponential definitions of cosh and sinh. The resulting expansions were generally well handled although there was the occasional error relating to the terms with the powers of either $A - B$ or $B - A$. However, obtaining the RHS was successfully achieved by the majority of the cohort. In part (b), the vast majority of candidates used the result from part (a) to expand $\cosh(x + \ln 2)$ and then substituted in the values of $\cosh(\ln 2)$ and $\sinh(\ln 2)$ to obtain an equation of the form $a \cosh x = b \sinh x$. From here, many obtained an equation involving tanh and then used the logarithmic equivalent form for $\operatorname{artanh} x$ to solve for x . Some of the responses, having expanded $\cosh(x + \ln 2)$, used the exponential definitions of both $\cosh x$ and $\sinh x$ to form an exponential equation and much success was achieved from this approach. A minority used the exponential definition of cosh to express, immediately, the LHS of the given equation in terms of exponentials which, having not used the result given in (a), scored no marks.

Question 2

In part (i) most candidates were able to complete the square of $5 + 4x - x^2$ but $(x - 2)^2 - 9$ and were seen. On integration the function $\arcsin f(x)$ was generally recognised and a high proportion of correct answers were seen. A few responses introduced $\arccos(\dots)$, $\operatorname{arsinh}(\dots)$ or $\operatorname{arcosh}(\dots)$.

In part (ii) despite being given a substitution of $x = 3 \sec \theta$ quite a number of candidates were unable to differentiate this and make any progress. The majority were able to complete the substitution and simplify to one of the forms $2 \cos \theta / \sin^2 \theta$, $2 \sec \theta / \tan^2 \theta$ or $2 \cot \theta \operatorname{cosec} \theta$. Many solutions faltered here as the integrand $k \operatorname{cosec} \theta$ was not recognised. An easy mark for changing the limits was not always achieved. Solutions which had correct integration generally substituted the limits correctly and reached the answer of $4 - 4\sqrt{3}/3$. $(12 - 4\sqrt{3})/3$ was seen but this did not fit the form shown on the question paper. One or two solutions curiously wrote $[-2/\sin \theta]$ with limits of $\sin \theta$ rather than θ but then achieved a correct answer.

Question 3

Part (a) was usually correct although only a minority were able to state the correct eigenvalue without any working but most candidates correctly set a solution up and obtained the correct eigenvalue.

Part (b) was well handled and understood with most candidates scoring both marks presenting their solution as described on the mark scheme. A small number of candidates started correctly, proceeded correctly to form at least 2 correct equations but then seemed unsure as to handle these correctly with a small number presenting their answer as $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$.

Part (c) appeared to differentiate well between candidates of differing abilities. The majority of candidates were able to form the correct Characteristic Equation. Most solved the CE using the Factor Theorem, correctly using the eigenvalues that had already appeared. There was some evidence of candidates using their calculators to solve the CE; this appears to be a growing trend as calculator technology advances. Having found a third eigenvalue, the vast majority of candidates were able to then form a correct diagonal matrix **D**, with the correct values following from their eigenvalues. The last 2 marks proved more challenging for a sizeable minority of candidates. Most were able to begin the process of forming the eigenvector but then were unable to handle the equations formed. The formation of the orthogonal matrix **P** was well handled by the majority of candidates, but a sizeable minority presented one of their columns as $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

Question 4

This question divided candidates who tended to score either zero or 4 marks. The most common method attempted was to apply the standard formula for differentiating $\operatorname{artanh}(\dots)$ together with the chain rule. It was quite common to see $\frac{dy}{dx} = \frac{1}{1 - \left(\frac{\cos x + a}{\cos x - a}\right)^2}$ which lost all marks. The quotient rule was generally well done though $\frac{d(\sin x)}{dx} = -\cos x$ did appear. Simplifying the above fraction was generally well done and combining with $2a \sin x / (\cos x - a)^2$ did produce many correct answers. Candidates who used product rule differentiation often found the algebraic simplification too tricky. A few candidates incorrectly wrote down the derivative of $\operatorname{artanh} x$ as $1 / (1 + x^2)$.

A small number of solutions started with $\tanh y = (\cos x + a) / (\cos x - a)$, then differentiated and attempted to rearrange. Weakness with the algebraic manipulation became a problem here for some candidates.

An unusual method chosen by a handful of candidates was to write $y = \frac{1}{2} \ln((1 + \dots)/(1 - \dots)) = \frac{1}{2} \ln(-\cos x/a)$. No consideration was given to the fact that $-\cos x/a$ could be negative. However, marks were credited for this method.

Question 5

The responses to this question generally started well with almost every candidate gaining the first 3 marks. Very few candidates were unable to correctly form the necessary derivatives and apply these to a correct formula for the surface area. The most challenging part of this question was how to handle the “Integration by Parts” on the te^t term. Most were able to perform the integration by parts correctly but were then let down by errors with signs when inserting this back into their integrand. Even those candidates who did not gain the second A or the B mark were still able to apply the limits correctly and give an answer in the form required.

Question 6

In part (a), the vast majority recognised the need to find the determinant of **A** and much success was achieved in finding it. Expansion along the top row of the matrix was, by far, the most popular way of finding $\det \mathbf{A}$ and the main reason as to why marks were lost here was sign errors. Many candidates then recognised the need to show that $\det \mathbf{A} \neq 0$ to prove the non-singularity of the matrix **A** and either writing the determinant in completed square form or using the discriminant were the main ways this was demonstrated. Much success was achieved here but then many of the cohort did not conclude that **A** is non-singular and thus lost the accuracy mark in this part.

In part (b), the well-rehearsed procedure of finding the inverse of a 3×3 matrix was clearly understood by nearly all candidates and thus much success was achieved. As usual, the main reason for the loss of marks in this process was due to sign errors occurring in forming the matrix of minors and/or the matrix of cofactors. Also, a significant minority of the candidates used, in finding \mathbf{A}^{-1} , a cancelled down (by 2) version of $\det \mathbf{A}$ which is clearly inaccurate and this lost such candidates two marks at the end of this part of the question.

Question 7

Candidates who noticed the need in part (a) to break x^n in to $x \cdot x^{n-1}$ used integration by parts correctly to get the correct first expression. Most of these candidates then correctly split $(10 - x^2)^{0.5}$ up so they could form two integrals and went on to gain full marks.

Candidates who did not notice the requirement to split x^n often went straight to integration by parts and tried to force this solution to give the required form. They did not go back to their first step to see why they were getting close to the solution but not the correct one. The discipline of including dx wasn't strong.

Part (b) was rarely laid out clearly. Most candidates were able to apply the reduction formula at least once. A number of students applied the formula twice but were unable to find I_1 , not noticing this required them to integrate. A surprising number involved I_2 and/or I_4 in error.

The majority of candidates did not progress to the correct numerical answer – with numerical and sign errors in the repeated application of the reduction formula causing most problems.

Question 8

The majority of candidates achieved full marks for part (a). Some of them had gave the coordinates in a column matrix. Only very few candidates used way 2. The majority of candidates progressed correctly to find the coordinates of Q .

In part (b) many candidates found two vectors in the plane PQR and the vector product between them but some of them did not find the constant using the scalar product between the part (a) vector and any of the position vectors of P , Q or R . There were various calculation errors when finding \mathbf{PQ} , \mathbf{RQ} and \mathbf{PR} , normally to do with not dealing with negatives correctly. Answers were left in various multiples of the form given in the mark scheme.

Part (c) was frequently not attempted or little progress was achieved. Some tried to find an equation for l_2 but did not attempt to find another point on l_3 . Those who did understand the question requirements generally progressed to full marks, but this was a rare sight!

Question 9

Part (a) provided an easy two marks for most candidates who knew to use the line equation to eliminate y from the ellipse equation and could then re-arrange the resulting equation to obtain the given result. Part (b) was more problematical. Most candidates used the quadratic formula to solve the equation given in (a). They then added their two solutions but many forgot that they needed to divide by 2. Many then used the line equation to find the y coordinate but failed to simplify their result.

Part (c) was omitted by many candidates, either because they did not know what to do or perhaps because they had run out of time. Some candidates used the ellipse equation instead of the equation given in this part of the question. Those who used the method of finding k in terms of x and y frequently were unable to achieve the required equation.